Rossmoyne Senior High School

Year 12 Trial WACE Examination, 2014

Question/Answer Booklet

MATHEMATICS:
SPECIALIST 3C/3D
Section One:
Calculator-free

SOLUTIONS

Student Number: In figures | | | | | | | | |

In words MARKING KEY

Your name ______________________

Time allowed for this section

Reading time before commencing work: five minutes
Working time for this section: fifty minutes

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet

To be provided by the candidate
Standard items: pens, pencils, pencil sharpener, eraser, correction fluid/tape, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be used in this section of the examination. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	7	7	50	50	$33\frac{1}{3}$
Section Two: Calculator-assumed	13	13	100	100	$66\frac{2}{3}$
			Total	150	100

Instructions to candidates

1. The rules for the conduct of Western Australian external examinations are detailed in the *Year 12 Information Handbook 2013*. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in the spaces provided in this Question/Answer Booklet. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.

3. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

4. It is recommended that you **do not use pencil**, except in diagrams.

Section One: Calculator-free (50 Marks)

This section has **seven (7)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time for this section is 50 minutes.

Question 1 (4 marks)

One of the solutions to $z^3 = c$ is $z = 3\sqrt{3} - 3i$.

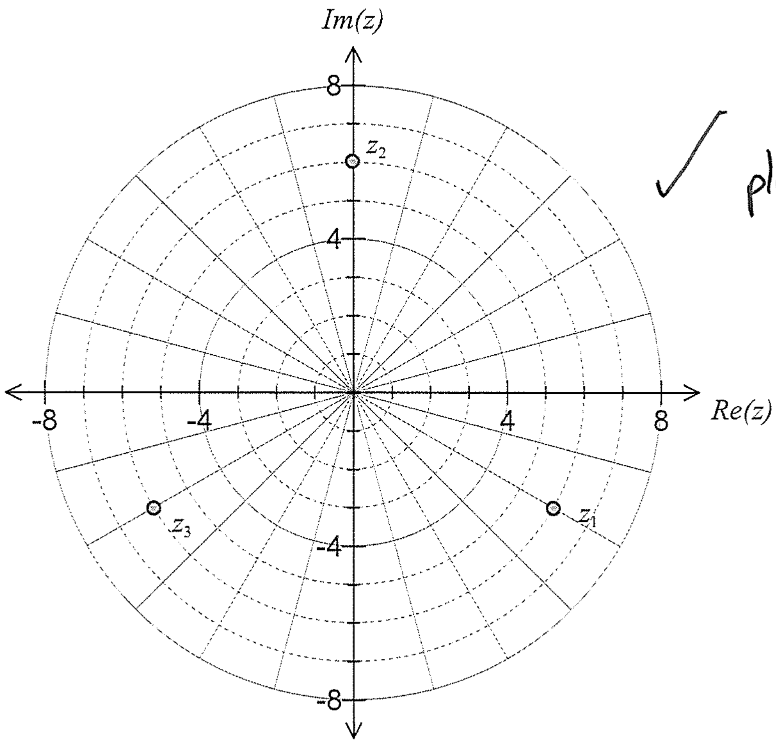
Determine all other solutions to the equation in Cartesian form and plot all solutions on the Argand diagram below.

$z_1 = 3\sqrt{3} - 3i = 6cis(-\frac{\pi}{6})$ ✓

$z_2 = 6cis(-\frac{\pi}{6} + \frac{2\pi}{3}) = 6cis(\frac{\pi}{2})$
$= 6i$ ✓

$z_3 = 6cis(-\frac{\pi}{6} - \frac{2\pi}{3}) = 6cis(-\frac{5\pi}{6})$
$= -3\sqrt{3} - 3i$ ✓

(−2 if given in polar only)



✓ plotted.

Question 2 **(8 marks)**

(a) If $f(x) = 2\sin x \cdot e^{\cos 3x}$, evaluate $f'\left(\frac{\pi}{6}\right)$. (4 marks)

$$u = 2\sin x\Big|_{x=\frac{\pi}{6}} = 1$$
$$u' = 2\cos x\Big|_{x=\frac{\pi}{6}} = \sqrt{3}$$

$$v = e^{\cos 3x}\Big|_{x=\frac{\pi}{6}} = 1$$
$$v' = -3\sin 3x e^{\cos 3x}\Big|_{x=\frac{\pi}{6}} = -3$$

$$f'(\tfrac{\pi}{6}) = \sqrt{3}\cdot 1 + 1\cdot(-3)$$
$$= \sqrt{3} - 3$$

✓ product rule
✓ Correct derivatives
✓ correct subs
✓ answer

(b) Use the substitution $u = \sqrt{2x+1}$ to evaluate $\int_0^4 \frac{e^{\sqrt{2x+1}}}{\sqrt{2x+1}}\,dx$. (4 marks)

$$u = \sqrt{2x+1} \Rightarrow du = \frac{1}{\sqrt{2x+1}}dx$$ ✓✓

When $x = 0,\ u = 1$ and $x = 4,\ u = 3$

$$\int_0^4 \frac{e^{\sqrt{2x+1}}}{\sqrt{2x+1}}\,dx = \int_1^3 e^u\,du$$ ✓
$$= \left[e^u\right]_1^3$$
$$= e^3 - e$$ ✓

Question 3 **(8 marks)**

(a) If $P=\begin{bmatrix}0 & 3\\-2 & 1\end{bmatrix}$ and $Q=\begin{bmatrix}1 & -1\\2 & -3\end{bmatrix}$ determine

(i) $2PQ$. (2 marks)

$$2\begin{bmatrix}0 & 3\\-2 & 1\end{bmatrix}\begin{bmatrix}1 & -1\\2 & -3\end{bmatrix}=2\begin{bmatrix}6 & -9\\0 & -1\end{bmatrix}=\begin{bmatrix}12 & -18\\0 & -2\end{bmatrix}$$

(ii) Q^{-1}. (2 marks)

$$\begin{bmatrix}1 & -1\\2 & -3\end{bmatrix}^{-1}=\frac{1}{-3-(-2)}\begin{bmatrix}-3 & 1\\-2 & 1\end{bmatrix}=\begin{bmatrix}3 & -1\\2 & -1\end{bmatrix}$$

(b) The matrix A, its inverse A^{-1} and a system of equations in a, b, c and d are shown below.

$$A=\begin{bmatrix}1 & 1 & 1 & 1\\1 & -1 & -1 & -1\\0 & 2 & 3 & 0\\2 & 0 & 0 & -1\end{bmatrix}\qquad A^{-1}=\begin{bmatrix}\frac{1}{2} & \frac{1}{2} & 0 & 0\\-\frac{3}{2} & -\frac{9}{2} & -1 & 3\\1 & 3 & 1 & -2\\1 & 1 & 0 & -1\end{bmatrix}\qquad \begin{aligned}a+b+c+d&=2\\2a-d&=3\\2b+3c&=6\\a-c-d&=b\end{aligned}$$

Using these and/or any other matrices, show use of matrix techniques to solve the system of equations. (4 marks)

$$\begin{aligned}a+b+c+d&=2\\a-b-c-d&=0\\2b+3c&=6\\2a-d&=3\end{aligned}\quad\Rightarrow\quad A\begin{bmatrix}a\\b\\c\\d\end{bmatrix}=\begin{bmatrix}2\\0\\6\\3\end{bmatrix}\quad\Rightarrow\quad\begin{bmatrix}a\\b\\c\\d\end{bmatrix}=A^{-1}\begin{bmatrix}2\\0\\6\\3\end{bmatrix}$$

$$\begin{bmatrix}\frac{1}{2} & \frac{1}{2} & 0 & 0\\-\frac{3}{2} & -\frac{9}{2} & -1 & 3\\1 & 3 & 1 & -2\\1 & 1 & 0 & -1\end{bmatrix}\begin{bmatrix}2\\0\\6\\3\end{bmatrix}=\begin{bmatrix}1\\0\\2\\-1\end{bmatrix}\Rightarrow a=1,\ b=0,\ c=2,\ d=-1$$

Question 4 **(8 marks)**

A small body moves along the x-axis so that after t seconds it is x centimetres from the origin. The velocity of the body for $t \geq 0$ is given by $v = \frac{x}{2} + 1$, where $x = f(t)$ and $f(0) = 0$.

(a) Show that $f(t) = 2e^{0.5t} - 2$. (3 marks)

$f(0) = 2e^{0.5(0)} - 2 = 0$

$v = \frac{dx}{dt} = f'(t) = e^{0.5t}$ ✓

✓

$x = f(t) = 2v - 2 \Rightarrow v = \frac{x}{2} + 1$ ✓

(b) Determine the acceleration of the body when $x = 4$ cm. (3 marks)

$x = 4 \Rightarrow v = 3$

$\frac{dv}{dt} = \frac{1}{2} \cdot \frac{dx}{dt}$ ✓

$= \frac{1}{2} \cdot v$ ✓

$= \frac{3}{2}$ cm/s^2 ✓

−1 units

(c) The body reaches a speed of 25 cm/s after $k \ln 5$ seconds. Determine the value of k. (2 marks)

$e^{0.5t} = 25$ ✓

$t = 2 \ln 5^2$

$= 4 \ln 5 \Rightarrow k = 4$ ✓

Question 5 **(5 marks)**

The production of a chemical in a laboratory can be modelled by the differential equation $\frac{dm}{dt} = e^{2t-m}$, where m kg is the total mass of the chemical produced after t hours.

Given that $m(0) = 0$, determine an exact value for the total mass of substance produced after three hours.

$$\frac{dm}{dt} = e^{2t} \cdot e^{-m}$$

$$\int e^{m}\, dm = \int e^{2t}\, dt \quad \checkmark$$

$$e^{m} = \tfrac{1}{2}e^{2t} + c \quad \checkmark$$

$$e^{0} = \tfrac{1}{2}e^{0} + c \Rightarrow c = \tfrac{1}{2} \quad \checkmark$$

$$e^{m} = \tfrac{1}{2}e^{2t} + \tfrac{1}{2}$$

$$m = \ln\left(\tfrac{1}{2}e^{2t} + \tfrac{1}{2}\right)\Big|_{t=3} \quad \checkmark$$

$$= \ln\left(\frac{e^{6}+1}{2}\right) \quad \checkmark$$

$$= \ln(e^{6}+1) - \ln 2 \text{ kg}$$

Question 6 **(10 marks)**

(a) Use Euler's formula $e^{ix} = \cos x + i\sin x$ to show that $\cos x = \dfrac{e^{ix} + e^{-ix}}{2}$. (3 marks)

$$\frac{e^{ix} + e^{-ix}}{2} = \frac{\cos x + i\sin x + \cos(-x) + i\sin(-x)}{2}$$
$$= \frac{\cos x + i\sin x + \cos x - i\sin x}{2}$$
$$= \frac{2\cos x}{2}$$
$$= \cos x$$

(b) Use the definition $\cos x = \dfrac{e^{ix} + e^{-ix}}{2}$ to show that $2\cos a \cdot \cos b = \cos(a+b) + \cos(a-b)$.
Do **not** use any identities from the formula sheet. (4 marks)

$$2\cos a \cos b = 2\frac{e^{ia} + e^{-ia}}{2} \cdot \frac{e^{ib} + e^{-ib}}{2}$$
$$= \frac{(e^{ia} + e^{-ia})(e^{ib} + e^{-ib})}{2}$$
$$= \frac{e^{i(a+b)} + e^{i(a-b)} + e^{i(-a+b)} + e^{i(-a-b)}}{2}$$
$$= \frac{e^{i(a+b)} + e^{i(-a-b)}}{2} + \frac{e^{i(a-b)} + e^{i(-a+b)}}{2}$$
$$= \frac{e^{i(a+b)} + e^{-i(a+b)}}{2} + \frac{e^{i(a-b)} + e^{-i(a-b)}}{2}$$
$$= \cos(a+b) + \cos(a-b)$$

(c) Show that $\int_0^{\pi/3} \cos 2x \cdot \cos x \, dx = \frac{\sqrt{3}}{4}$. (3 marks)

$$\int_0^{\pi/3} \cos 2x \cdot \cos x \, dx = \frac{1}{2}\int_0^{\pi/3} \cos 3x + \cos x \, dx$$

$$= \frac{1}{2}\left[\frac{\sin 3x}{3} + \sin x\right]_0^{\pi/3}$$

$$= \frac{1}{2}\left(\left(0 + \frac{\sqrt{3}}{2}\right) - (0+0)\right)$$

$$= \frac{\sqrt{3}}{4}$$

Question 7 **(7 marks)**

A small rocket is launched from position $8\mathbf{j}+2\mathbf{k}$ on a school oval and moves with a constant velocity of $12\mathbf{i}-4\mathbf{j}+8\mathbf{k}$. At the same instant the rocket is launched, a small balloon at position $20\mathbf{i}-10\mathbf{j}+a\mathbf{k}$ is being blown horizontally by the wind with constant velocity $4\mathbf{i}+4\mathbf{j}$. All distances are in metres and all velocities are in metres per second.

The rocket misses the balloon, with the distance between them a minimum exactly two seconds after its launch.

(a) Determine the value of the constant a. (5 marks)

$$_R\mathbf{r}_B = \begin{bmatrix}0\\8\\2\end{bmatrix} - \begin{bmatrix}20\\-10\\a\end{bmatrix} = \begin{bmatrix}-20\\18\\2-a\end{bmatrix} \quad ✓ \qquad _R\mathbf{v}_B = \begin{bmatrix}12\\-4\\8\end{bmatrix} - \begin{bmatrix}4\\4\\0\end{bmatrix} = \begin{bmatrix}8\\-8\\8\end{bmatrix} \quad ✓$$

When $t=2$, $_R\mathbf{v}_B \bullet (t \times {}_R\mathbf{v}_B + {}_R\mathbf{r}_B) = 0$ ✓

$$\begin{bmatrix}8\\-8\\8\end{bmatrix} \bullet \begin{bmatrix}2\times 8-20\\2\times -8+18\\2\times 8+2-a\end{bmatrix} = 0 \quad ✓$$

$$8(-4-2+18-a)=0$$
$$12-a=0$$
$$a=12 \quad ✓$$

(b) Determine the minimum distance between the rocket and the balloon. (2 marks)

$a=12 \Rightarrow 2\,{}_R\mathbf{v}_B + {}_R\mathbf{r}_B = -4\mathbf{i}+2\mathbf{j}+6\mathbf{k}$ ✓

$|-4\mathbf{i}+2\mathbf{j}+6\mathbf{k}| = \sqrt{16+4+36}$
$=\sqrt{56}$
$=2\sqrt{14}$ metres ✓

Additional working space

Question number: _________

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